Baseball Bat Regulations: Preserving America's Pastime and our Love for Mathematics

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Abstract

In the game of baseball, a good batter knows how to consistently transfer as much energy to the ball as possible. In order to accomplish this task with a wooden bat, a player attempts to hit the ball in the "sweet spot" of the bat, a region that minimizes the resulting vibrations. By reducing these vibrations, less energy is lost to the bat, and more is imparted to the ball. In an attempt to outperform their fellow players, some have performed the practice known as "corking" to increase their batting results. These players believe that "corking" increases the "sweet spot effect." However, in our analysis, we find that there is a tradeoff between the loss of mass in the bat and the faster swing speed and increased swing control. In fact, it has been suggested that the improvements seen with corked bats are more psychological than physical. Additionally, the use of bats made of other materials is prohibited in Major League Baseball. Our findings show that this is because of increased energy transfer efficiency in the bat-ball interaction caused by the difference in the fundamental frequency of the vibrations in different materials. In other words, since a composite or aluminum bat naturally vibrates at a higher frequency than one made of wood, it will provide more energy to the ball. In hollow bats, the addition of a phenomenon known as the "trampoline effect" further increases the transfer efficiency through high frequency vibrations in the shell-shaped surface of the bat. Based on our analysis, using wooden bats is the best way to ensure that baseball remains a game based on the skill of its players.

Contents

1 Introduction

Experienced baseball players are familiar with the concept of a "sweet spot" in a baseball bat. The idea is that by using a certain region of the bat to make contact with the ball, the transfer of energy to the ball is maximized, resulting in a superior hit. This would mean that knowledge of the location of this "sweet spot" would enhance the hitting ability of a player.

The first step is to verify the science behind the sweet spot, specifically in regards to the idea of torque. The basic principles of torque would indicate that the most powerful hit would be achieved by hitting the ball with the very end of the bat. However, evidence indicates that this is not correct with a baseball bat. Thus, we will determine the location of the sweet spot based on the principles that govern the effect.

Throughout the history of baseball, the question of "corking" has been a highly controversial issue. Some players believe that by drilling a hole in the end of the bat and replacing the wood with a lighter material, it is possible to improve hitting performance through the increased swing speed. The next step in our analysis will be to determine the factual basis for this claim and to find the actual effects of corking on hitting quality.

Finally, the use of aluminum bats is prohibited in Major League Baseball. It would make sense that this ban coincides with an increase in performance from these metal bats. The question, however, is what causes this effect? In order to determine the reason for the ban on metal bats, we will explore the effects of the material on the sweet spot effect.

2 Modeling a Typical Wooden Bat

The concept of the sweet spot relies on the vibrations that occur within the bat after it has struck the ball. Depending on the point of impact, the bat will oscillate in one or more of several "modes," related to the natural harmonics of the bat.

The waves in the bat generated by the impact are standing waves, meaning that the nodes and anti-nodes remain stationary as the bat oscillates orthogonal to the length of the bat.

In order to make a simple model of the oscillations, we make the following assumptions:

1. We treat the bat as a perfect cylinder. This allows us to ignore the fluctuations in amplitude that would result from variations in stiffness caused by the changing radius of the bat.

2. We treat the collision between ball and bat as a one-dimensional impact rather than two- or three-dimensional. This is important for the sake of simplicity since the spin of the ball before the collision or even the spin caused due to a glancing blow by the bat affects the interaction between the ball and the bat.

3. We assume that all wooden bats of all sizes act in a similar manner. The regulations for professional bats allow for a certain degree of choice in length and weight. While our model considers the length, this simple model ignores different weights (and hence different thicknesses to an extent).

With these ideas in place, we can begin to construct our model for a wooden bat.

2.1 Clamped vs. Freestanding Bat

The most simple and intuitive model to use for a baseball bat is to have a pipe with one end fixed to represent the batter's grip on the bat. However, studies have shown that this is not true. For example, an experiment by Professor Howard Brody has shown that by observing the vibrations of a hand-held bat, it can be shown that the bat behaves as if it were a free body at the impact of the bat and the ball [1].

One of the central differences between a clamped and free bat is the allowable normal modes of oscillation of the bat. For a baseball bat clamped at one end, the lowest mode of oscillation present would be the fundamental mode with a node at the clamp and an anti-node on the other end of the bat. In addition, the next higher mode would also be present, which is the first harmonic mode with one node at the clamp and another somewhere beyond the center of mass.

For a free body baseball bat, the fundamental oscillation is not possible, and the lowest frequency oscillation would consist of two nodes. Figure 1 from Professor Brody's experiment show a graphical depiction of the lowest modes of oscillation of the clamped and free bats.

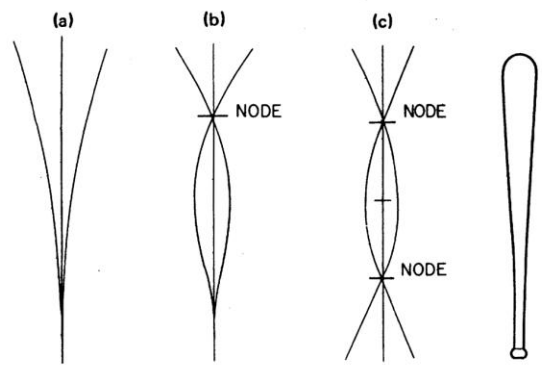

Figure 1. Oscillations of a baseball bat with, (a) and (b), one end clamped and, (c), both ends free [1]

The experiment consisted of hitting the baseball bat at an anti-node of the first harmonic (i.e. the tip of the bat) and observing the resultant frequencies present in the vibrations of the bat. If a hand-held baseball bat showed the low fundamental mode frequency, then it would be a good indicator that the clamped model is more accurate. However, if the fundamental mode were not present, then it would indicate that a free body model is more accurate.

Professor Brody's report concludes that the free-body model is the most accurate representation of vibrations resulting from an actual hand-held bat to ball impact. This is most likely because whatever force a batter's grip could exert on the bat is insignificant compared to the enormous amount of force exerted by the ball on impact. However, he did discover that the batter's grip affected the rate at which the vibrations damp out, with a tighter grip resulting in much faster damping.

An interesting point regarding these findings is that if the batter released the bat precisely as it impacted the ball, the transfer of energy, and hence resulting path of the ball, theoretically would remain unchanged from if the batter held onto the bat. This would of course create a dangerous situation, particularly for the pitcher, as he would now find himself in the trajectory of the bat.

2.2 Location of the Sweet Spot

Now that we have determined that a freestanding bat is the more accurate model, we can begin to determine the location of the sweet spot in a standard wooden bat.

A ready comparison that can be made to our model is that of an open-ended pipe and the standing sound waves inside it. Since both ends of the bat are free, they correspond to the open ends of the pipe.

The location of the sweet spot, based on our model, relies solely on the length of the bat. In our theoretical cylindrical bat, the first vibration mode corresponds to the second harmonic, shown in Figure 2a, and the second mode corresponds to the third harmonic, shown in Figure 2b. In the figure, the bat handle is considered to be on the right hand side.

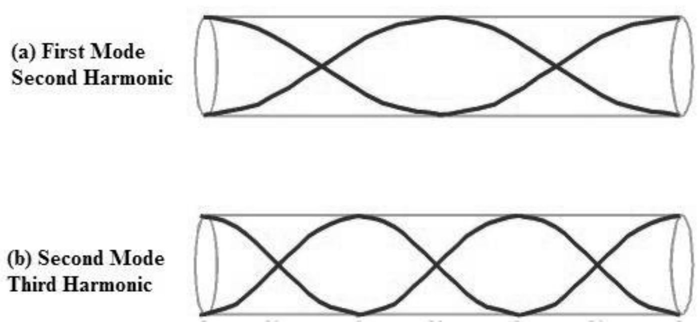

Figure 2. Vibration Modes as Standing Waves [2]

The intersections of the two waves displayed in each pipe are nodes, meaning that the interference pattern causes no vibration at that point. The areas of maximum displacement between the waves are anti-nodes, resulting in the greatest amount of vibration amplitude experienced there.

- **The sweet spot is the region between the nodes for the first and second vibration mode that occur farthest from the handle of the bat** [3]**.**

In the first vibration mode, the wavelength of the vibration in relation to the length of the bat is given in Equation 1 below, where $\boldsymbol{L}$ is the length of the bat and $\boldsymbol{\lambda}$ is the wavelength of the oscillation. This is the case because, as shown in Figure 1a, exactly one complete wave of is present in the length of the bat.

$$\boldsymbol{\lambda = L} \tag{1}$$

For the second mode, one and a half waves are present in the bat, so Equation 2 below relates the wavelength to the bat length.

$$\boldsymbol{\lambda = \frac{2L}{3}} \tag{2}$$

In both cases, the farthest node is one-quarter wavelength from the end of the bat. If we make a substitution for this fact into Equations 1 and 2, we can develop Equation 3 to give us the range for the location of the sweet spot.

$$\frac{L}{6} \; to \; \frac{L}{4} \; from \; the \; end \; of \; the \; bat \tag{3}$$

For example, in a 33 inch bat (a common length), the sweet spot would be between 5.5 and 8.5 inches from the end of the bat.

The Major League does not mandate a specific bat size other than a limit of 42 inches long and no more than 2.75 inches in diameter at the widest part. Additionally, the weight of the bat must be at least the value of the length minus 3 ounces. Thus, we can use Equation 3 to determine the sweet spot zone for any bat given its length.

In reality, the shape of the bat would shift the actual location of the sweet spot toward the hitting end of the bat due to the changes in stiffness as the bat diameter changes. However, based on our simplified model, this will provide a general region for the sweet spot. Section 6 provides an exploration of changes to the model that can more accurately describe this idea.

By striking the ball in this region of the bat, the batter is able to minimize the energy lost to vibrations in the bat and increase the amount of energy transferred to the ball, resulting in a stronger hit.

3 Parameters that Affect Ball-Bat Collision

This section explores the other factors that govern the energy transfer during the collision between the ball and bat.

3.1 Coefficient of Restitution

The coefficient of restitution (COR) is a measurement of the inefficiency inherent in the interaction between a baseball and the bat. This inefficiency is because the ball undergoes a large amount of distortion during collision and much of the original kinetic energy is dissipated in the process [4]. The COR of a baseball is determined by firing a baseball at approximately 60mph (26.6m/s) against a flat wall and calculating the ratio between the relative velocity after the collision to that before the collision [5]. It is represented by the letter ***e*** as shown in equation 4 below.

$$\boldsymbol{e} = \frac{v_{after}}{v_{before}} \tag{4}$$

The COR can also be used to determine the proportion of the original kinetic energy of the ball lost in the collision through equation 5. For example, with a COR of $\boldsymbol{e} = 0.5$, there will be a $1 - (0.5)^2 = 0.75 = 75\%$ loss of the original kinetic energy

$$1 - \boldsymbol{e}^2 = proportion\ of\ original\ kinetic\ energy\ lost \tag{5}$$

As previously mentioned, the COR of a baseball is determined by its collision with a flat wall. We will hereafter refer to this COR as $\boldsymbol{e_0}$. However, the COR of the collision between a baseball and a bat will be different as a baseball bat is flexible which results in both the ball and the bat undergoing some compression during collision. Therefore, some of the energy loss from the original kinetic energy of the ball is converted into compressional energy in the bat.

The COR of the ball-bat collision, ***e***, is dependent on how efficiently the compressional energy transferred to the bat is returned to the ball. For a wooden bat, most of that energy is translated into low-frequency bending vibrations [4]. As a result, *e* for a wooden bat will never exceed $\boldsymbol{e_0}$.

In the case of a hollow bat, such as the popular aluminum bat or a composite bat, the energy is also stored in hoop modes, which will be explained in detail in the next section. The energy stored in hoop modes is returned efficiently to the ball, which causes ***e*** to exceed $\boldsymbol{e_0}$. This phenomenon is called the "Trampoline Effect."

3.2 Trampoline Effect

In addition to the sweet spot created by the harmonic vibrations of the bat, hollow bats experience an additional trait known as the trampoline effect. It results from vibrations in the surface of the bat. The simplest way to model this phenomenon is to consider the bat as a hollow cylinder.

According to thin-shell theory, Equation 6 below can describe the stiffness, ***k***, of our model.

$$\boldsymbol{k} \propto \left(\frac{t}{r}\right)^3 \tag{6}$$

Where ***t*** is the thickness of the shell and ***r*** is its radius. This tells us that the Trampoline Effect is negligible in the handle (since the cubic of the radial difference weights it substantially), allowing us to focus the model on the fat portion of the bat.

The stiffness relates to the ability of the bat to vibrate. Just as in the case of a spring constant, ***k*** is a relationship between force and displacement. That is to say, the stiffness, ***k***, is directly proportional to the amplitude of the vibrations in the bat. In the trampoline effect, this means that the lower the ***k*** value is, the more pronounced the trampoline effect.

If one listens to the sound of a wooden bat and a metal bat, the difference is obvious. While a wooden bat results in a "crack," a metal bat releases a notable "ping." By attaching a microphone to the handle of a bat and striking a ball near the sweet spot, Dr. Daniel Russell of Kettering University developed a plot of the impact sound, seen in Figure 3 below.

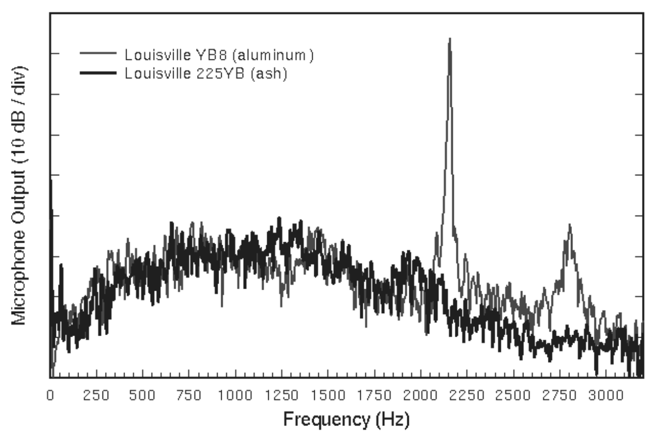

Figure 3. Sound Output from Wooden vs. Aluminum Bat [6]

In the plot, the peaks around 2200 Hz and again at 2800 Hz are easily noticeable. The peak at 2200 Hz corresponds to the first "hoop" vibration mode. In this mode, the barrel alternately contracts and expands. The second mode, at 2800 Hz, has a hoop nope, similar to the nodes in the normal bending modes, where no motion occurs. In fact, the location of this node is usually very close to the sweet spot of the normal bending modes [6].

Thus, a hit near the sweet spot an aluminum bat creates the signature “ping” because the higher frequency vibrations create a higher frequency sound. However, at this point, while the first hoop mode is highly active, the second mode is nearly nonexistent. Because of this fact, the real advantage of the aluminum bat is when the hit occurs away from the sweet spot. Whereas the bending modes in a wooden bat will absorb some of the energy, an aluminum bat will return a portion of this energy to the ball through the hoop modes. The efficiency of the energy transfer is explored in further detail in Section 5.1.

4 “Corking”

Some baseball players believe that “corking” the bat produces an enhanced sweet spot effect and improves their performance. Corking is a process where a hole, approximately 1 inch in diameter, is drilled into the end of the baseball bat between 6 to 10 inches deep and filled with a lighter substance such as cork. This results in a lighter bat and moves the center of mass closer to the handle, resulting in a faster and more easily controlled swing [7].

It is not clear, however, whether or not corking the bat improves performance. Having a corked bat moves the center of mass of the bat closer to the handle, reducing the moment of inertia and allowing a batter to swing the bat faster. On the other hand, it also decreases the weight of the bat, reducing the effectiveness of the ball-bat collision. Whether or not corked bats hit ball further or faster is a tossup between these tradeoffs.

- **Faster and more controlled swing**

This advantage can be derived from the basic relationship between control of swing and mass. Anyone can quickly test to see that it is easier to control the swing of a lighter object. Similarly, corking the bat decreases the mass of the bat and allows for a more controlled swing. For an amateur player the mass difference may not make too much of a difference, but for a professional player it could provide that slight advantage to tip the game.

With a lower mass, there is also a lower moment of inertia [8]. In basic terms, the moment of inertia is the measure of an object’s resistance to changes in its rotation rate. Consequently, a baseball player would rather have a bat with a lower inertia so that he can swing it faster. Physics tells us that the moment of inertia can be calculated as follows:

$$\boldsymbol{I} = \frac{1}{2}\boldsymbol{m} \cdot \boldsymbol{r}^2 \qquad (7)$$

Where ***m*** is the mass of the bat and ***r*** is the rotation vector. Equation 7 shows that mass is proportional to the moment of inertia, as is the square of the radius to the center of mass. Corking decreases mass *and* moves the center of mass closer to the handle, decreasing ***r*** as well. This causes the moment of inertia to decrease allowing a batter to swing the bat faster.

Another explanation as to why a faster swing is an advantage is through a very simple concept. The faster the batter is able to swing the bat, the longer he can delay until making a decision to swing. Although it seems that such a slight difference in mass would not make a significant difference, a professional baseball player would be able to watch the ball travel approximately an additional 5-6 feet before having to commit to a swing [8]. The relationship can be modeled as follows:

$$\frac{1}{mass} \propto speed\ of\ swing \propto time\ to\ commit\ to\ swing \qquad (8)$$

- **Less Effective Collision**

A lower mass bat also has a negative consequence for the batter. If the swing speed is unchanged, then a heavier bat will always hit the ball faster and farther. Therefore lowering the mass of the bat will result in a less effective collision.

$$m_1 v_1 + m_2 v_2 = m_1 v_3 + m_2 v_4 \qquad (9)$$

In other words, from the simple momentum conservation equation above, if $\boldsymbol{v_1}$, $\boldsymbol{v_3}$, $\boldsymbol{m_2}$, and $\boldsymbol{v_2}$ remain unchanged but $\boldsymbol{m_1}$ decreases, then $\boldsymbol{v_4}$ must decrease as well to conserve momentum. Therefore, the batted-ball speed will decrease, which is disadvantageous for a batter.

Empirical evidence suggests that corking a bat actually reduces the performance of a bat. As shown in Figure 4, corking a bat actually reduces the COR of the ball-bat collision. Unsurprisingly, it also reduces the durability of the bat. For example, L.P. Fallon and J.A. Sherwood discovered in their experiment that corked bats cracked after as few as three impacts [8].

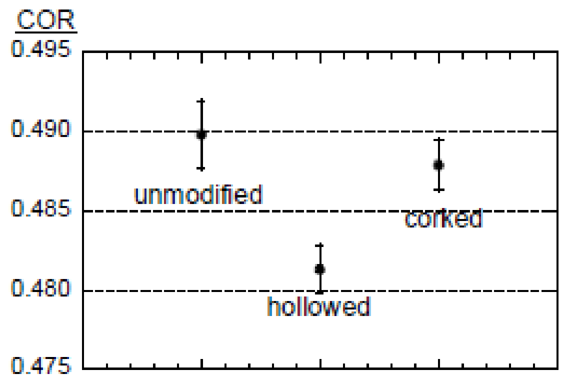

Figure 4. Measured COR for unmodified, hollowed, and corked bats [4]

Some batters also claim that corked bats give rise to the trampoline effect. However, the trampoline effect caused by corking would be quite insignificant. The trampoline effect is most prominent in aluminum bats, which are not allowed in professional baseball leagues, so for this example we will use regulations for college baseball, which requires a maximum of $2^5/_8" = 2.625"$ in diameter for aluminum bats. For simplification purposes, we will use two bats, 2.6" in diameter. The thickness of a typical aluminum bat wall is about 0.1" whereas the corked bat with a 1" hole would have $\frac{2.6"-1"}{2} = 0.8"$ thick walls. Using Equation 6, the ratio between the stiffness of the two bats would be

$$\left(\frac{0.1}{2.6}\right)^3 : \left(\frac{0.8}{2.6}\right)^3 \rightarrow 1:512, \qquad (10)$$

showing that the corked bat is 512 times more "stiff" than the aluminum bat. Therefore, the trampoline effect in a corked bat is quite insignificant compared to that on an aluminum bat.

Another explanation suggested about the effect of corking is that it produces more of a psychological effect on the batter than an actual physical one. A batter using a corked bat would *feel* as though he will perform better, so he may actually perform better due to a placebo-like effect [9].

As shown above, corking does not provide a clear-cut advantage or disadvantage to the batter. So why is corking banned in the professional leagues? The fact of the matter is that a corked bat has the possibility of giving an unfair advantage, and thus corked bats are banned in order to remove this possibility. In addition, allowing baseball games to run within set boundaries allows professional leagues to preserve the meaning of baseball tradition.

5 Effects of Bat Material

The material of which a bat is composed also affects a batter's performance. It is one of the main characteristics in finding relevant properties such as mass and stiffness [10]. Once these properties are found it is possible to find the natural frequency, which is directly proportional to the energy transfer to the baseball from the bat. Not only does natural frequency of the bat affect the batter's performance, but also the hollowness of the bat [11].

The hollowness of the bat relates to what we call the Trampoline Effect (explained in Section 3.2) [12]. Both of these aspects of the material of a bat are what are to be considered and consequently explored when examining batting performance resulting from Wood, Aluminum, and other material bats.

5.1 Aluminum Bats

In order to analyze the material of the aluminum bat versus the wooden bat and its effects on the batter's performance, the natural frequency must be found of the two different bats. The reasoning behind finding the natural frequency, $\boldsymbol{f}$, is that it is directly proportional to the COR, $\boldsymbol{e}$.

$$\boldsymbol{f} \propto \boldsymbol{e} \tag{11}$$

One way to model the bat-ball interaction is a type of spring mass system. The ball is represented by a mass with a spring attached to the impact side, while the bat is represented as a mass attached to a fixed point by a spring. The spring constants for these springs are related to their stiffness, thus $\boldsymbol{k}$ is used as the variable for stiffness in the model. Figure 5 below is a graphical representation of the spring-mass model.

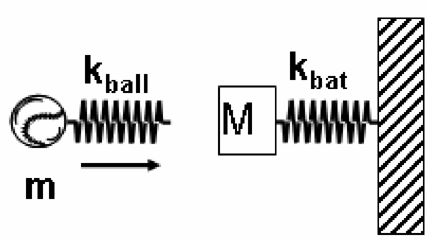

Figure 5. Spring-Mass Model of Bat-Ball Interaction[4]

By following this model, in order to find the natural frequency, the following equation is used taking into account, the stiffness, $\boldsymbol{k}$, and the mass, $\boldsymbol{m}$ [11]:

$$\boldsymbol{f} = \frac{\mathbf{1}}{\mathbf{2}\boldsymbol{\pi}}\sqrt{\frac{\boldsymbol{k}}{\boldsymbol{m}}} \tag{12}$$

We use an example experiment with wooden and aluminum bats to acquire the mass of the material (where American White Ash wood's mass is $\boldsymbol{m_{wood}}$ and aluminum's mass is denoted as $\boldsymbol{m_{aluminum}}$):

$\boldsymbol{m_{wood}}$ = 0.846 kg [1]
$\boldsymbol{m_{aluminum}}$ = 0.825 kg [1]

However, we must go through a process in order to find the stiffness value. In order to find the stiffness, the scenario should be explained of which the ball's force, $\boldsymbol{F_{app}}$, is applied to the bat resulting in a displacement of the bat, $\boldsymbol{\Delta y}$. Figure 6 is a representation of the scenario:

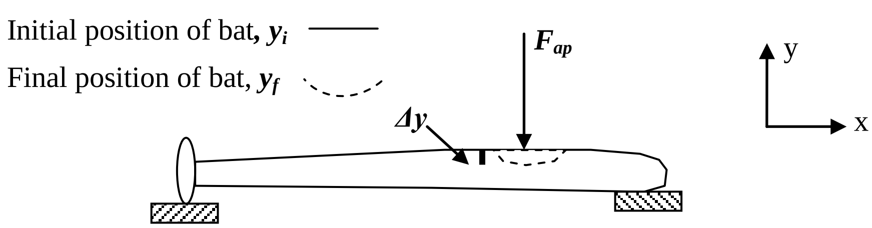

Figure 6. Scenario Explanation of Stiffness

By using data collected in an experiment by another party, we were given the following values for $\boldsymbol{F_{app}}$ and $\boldsymbol{\Delta y}$ for both American White Ash Wood and Aluminum bat scenarios. In the experiment, both ends of the bat were fixed and a force was applied to the bat. The resulting displacement of the bat was then measured.

Material	$\boldsymbol{F_{app}}$ (N)	$\boldsymbol{\Delta y}$(m)
Wood	98 N	4.2×10^{-4}
Aluminum	98 N	1.5×10^{-4}

Table 1. Applied Force and Displacement of Bat in Experiment [1]

Subsequently, the following equation for $\boldsymbol{k}$ is utilized:

$$\boldsymbol{k} = \frac{\boldsymbol{F_{app}}}{\Delta \boldsymbol{y}} \tag{13}$$

By substituting in the values given in Table 1, it is now possible to find the wood's stiffness, $\boldsymbol{k_{wood}}$, and the aluminum's stiffness, $\boldsymbol{k_{aluminum}}$.

$$\boldsymbol{k_{wood}} = \frac{98 \text{ N}}{4.2 \text{ x } 10^{-4} \text{ m}} = 2.33 \text{ x}10^5 \text{ N/m} \tag{14}$$

$$\boldsymbol{k_{aluminum}} = \frac{98 \text{ N}}{1.5 \text{ x } 10^{-4} \text{ m}} = 6.53 \text{ x}10^5 \text{ N/m} \tag{15}$$

Now that both the $\boldsymbol{k}$ and $\boldsymbol{m}$ values have been acquired, the fundamental frequency of the aluminum bat, $\boldsymbol{f_{wood}}$, and the wooden bat, $\boldsymbol{f_{aluminum}}$, can now be calculated:

$$\boldsymbol{f_{wood}} = \frac{\boldsymbol{1}}{\boldsymbol{2\pi}}\sqrt{\frac{2.33 \text{ x}10^5}{0.846}} = 83.5 \text{ Hz} \tag{16}$$

$$\boldsymbol{f_{aluminum}} = \frac{\boldsymbol{1}}{\boldsymbol{2\pi}}\sqrt{\frac{6.53 \text{ x}10^5}{0.825}} = 141.6 \text{ Hz} \tag{17}$$

The fundamental frequency of the bat is related to the efficiency of the energy transfer to the ball. Because the ball is in contact with the bat for such a short time period, the higher the frequency, the more energy the bat transfers to the ball.

Equation 18 below demonstrates the relationship between the vibration frequency, $\boldsymbol{f}$, and the period of that frequency, $\boldsymbol{t}$. Additionally, the variable, $\boldsymbol{\tau}$, is the period of contact between the ball and the bat.

$$\boldsymbol{t} = \frac{1}{\boldsymbol{f}} \tag{18}$$

Because of this relationship, when the frequency of vibration causes a period that is shorter than the contact period ($\boldsymbol{t} < \boldsymbol{\tau}$), the vibrations return some of the energy dissipated to create them back into the ball. However, as we have explored, the frequency of the bending vibration modes are very low, resulting in a period in the 10 ms range. Since this is so much longer than the 1 ms impact time, the ball does not experience a full cycle of vibration.

In contrast, the frequency of the hoop modes generated by the trampoline effect (Section 3.2) in hollow bats is much higher, resulting in periods of less than one millisecond. Thus, the hoop modes return some of their energy back to the ball, creating a stronger hit.

This also affects the COR for the impact, causing to $\boldsymbol{e}$ be greater than $\boldsymbol{e_0}$, meaning that the exit velocity of the ball is higher for an aluminum bat.

6 Improving the Model

In our initial model of a wooden bat, we treated it as a perfect cylinder. While this simplified our calculations for finding the location of the sweet spot, it is not completely accurate.

The stiffness of the bat, $\boldsymbol{k}$, is proportional to the radius of the bat, $\boldsymbol{r}$, as shown in Equation 19 below.

$$\boldsymbol{k} \propto \boldsymbol{r}^4 \tag{19}$$

This means that the amplitude of the vibrations in the barrel will be significantly lower than the amplitude in the handle. Rather than the simplistic plots of the simple harmonics used in Section 2.2, the actual vibrations will look like Figure 7 below.

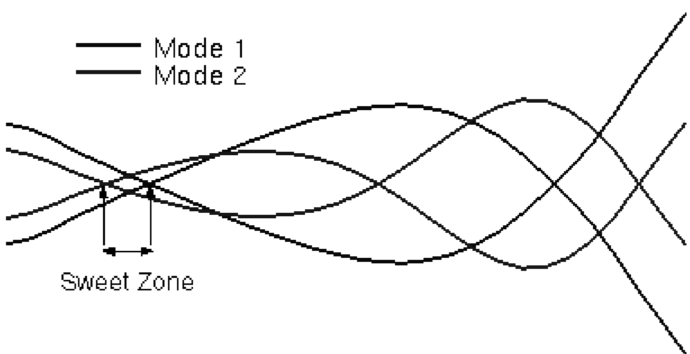

Figure 7. Harmonic Plot in Actual Bat [3]

Since the handle allows for greater amplitude, most of the energy transferred to the bat from the ball becomes stored in the handle vibrations. This is why the transfer efficiency is so low. In order to model a real bat accurately, an equation would be created that relates the energy transfer efficiency (COR) to the stiffness as it changes in relation to bat radius.

After examining multiple images of baseball bats, we decided to use the logistic equation for the cumulative distribution function to model the radius of a bat. Combining image analysis with measurements of actual bats and data that we collected, we constructed a model for the radius with respect to the location along the bat, given in Equation 20 below.

$$\boldsymbol{r}(\boldsymbol{t}) = \frac{3}{4}\left(1 - \frac{1}{1+e^{\frac{(0.368\,\boldsymbol{L})-\boldsymbol{t}}{(0.074\,\boldsymbol{L})}}}\right) + \frac{1}{2} \tag{20}$$

Where ***t*** is the displacement from the barrel end of the bad and ***L*** is the total length of the bat. This function is valid as long as ***L*** remains in the regulation range. If we combine Equations 19 and 20, we would now have a function, which we will call ***k(t)***, which represents the stiffness of the bat as it changes along the length of the bat. Plots of both ***r(t)*** and ***k(t)*** appear in Appendix A.

The next step of this method would be to treat this function, ***k(t)***, as a variable damping function in a spring-mass-type harmonic motion model. However, the computational rigor of this task is not proportional to the additional accuracy to the location of the sweet spot.

Another possible approach to modeling the situation is what Dr. Russell, whose work has formed a fair portion of our analysis, calls a “dynamic model.” In essence, the bat is treated as a non-linear spring experiencing vibrations. This is then modeled using a partial differential equation, which can be explored for different aspects of the system [13]. However, as in the previous option, the mathematical intricacy does not seem to result in a sweet spot location that is noticeably different from our original answer. The real benefit of this particular model stems from the applicability to comparing various types of bats.

Additionally, one limitation to our original model is that it does not consider the displacement of the ball in the third spatial dimension during impact; we assumed that each collision was directly head on with no variance in height. Hitting low on the ball creates backspin as the ball flies, while hitting high forces the ball downward toward the ground. This spin reduces the amount of energy propelling the ball in its trajectory since only a certain amount of energy exists in the ball, and translational and rotational kinetic energy must be conserved.

7 Conclusions

The game of baseball is an American past time. While the basic rules of game play have not changed, it is easy to see how the circumstances surrounding the game have. Specifically concerning baseball bats, new technology continues to arise that changes the face of baseball.

We initially addressed the idea of the “sweet spot” of a bat, a region wherein a hit results in the greatest possibly energy transfer. We considered the impact of the vibrations generated in the bat on this transfer of energy. Our initial model only considered wooden bats, the only type that is authorized in the major leagues. Using a simple harmonic model, we found that the sweet spot is consistently between one sixth and one quarter of the full length of the bat.

Moving beyond the simple wooden bat, we explored the problem of “corking,” where a player alters a wooden bat in an attempt to improve performance. Our analysis indicated that both mathematically and empirically, it is difficult to place a significant advantage or disadvantage of corking. We thus concluded that the idea behind the ban on corking is that it removes the *possibility* of an unfair advantage in order to preserve the spirit of baseball.

Another consideration regarding corking is an attempt to create the trampoline effect. When a hollow bat is used (specifically metal), additional higher frequency vibrations in the surface of the bat increase the energy transfer efficiency. However, because the walls of a corked bat are so much thicker than a metal bat, we found that the trampoline effect is negligible.

In contrast, the trampoline effect provides a significant advantage to metal and composite bat. The high frequency hoop vibration modes increase the energy returned to the ball, resulting in faster return speeds and thus farther distance. In the same way that corking is banned from Major League Baseball, so are metal and composite bats.

The most logical explanation for these bans is a desire of the League to maintain the fairness of baseball as a whole. Despite efforts like these, the baseball community continues to be the home of frequent cheating scandals, sometimes for bat problems. When Barry Bonds broke the home run record, he did so using bats made of Maple wood, rather than the tradition American White Ash. The League subsequently reviewed this fact to determine if the material gave him an unfair advantage.

In order to maintain the legitimacy of baseball, close regulation of bats is vital.

7.1 Further Considerations

The time limitations of this competition did not allow us to explore all aspects of this problem. Our models create a general representation of performance resulting from different types of baseball bats, but certain factors reduce the overall accuracy to be expected:

- The stiffness of a bat's material, as well as the motion of the bat and ball, can be influenced by uncontrollable variables such as temperature and humidity of weather. For instance, the humidity would affect the characteristics of wood while higher temperatures affect the characteristics of metal (in this specific case, aluminum).

- Some of our values of data were only confirmed by one source, not multiple sources. Given more time, we could have considered a broader base of knowledge than we did. This would have given us greater background information as well as additional ideas for modeling methods.

- We often used the results of experiments as a basis for our results. Given enough time and resources, it would have been possible to design and conduct experiments of our own to either confirm or refute our research and add additional legitimacy to our own results.

- Our models ignored the multi-dimensional nature of the bat-ball interaction. More time would allow us to consider the actual energy transfer in three-dimensions, resulting in a more accurate analysis of the sweet spot beyond a lateral location, but also a vertical location on the bat.

Regardless of these concerns, our analysis indicates a significant difference in the different types of bats and their resulting performance. Perhaps the main further consideration to be explored is the possibility of making baseball bats that can counteract the variables of the weather such as temperature and humidity.

Appendix A Additional Model Plots

This appendix provides the plots generated by the model described in Section 6, which is, using a function to describe the radius of a bat. We first developed a function for the radius in terms of the total length of the bat and the displacement along it, ***r(t)***, which is reproduced in Equation A1 below.

$$\boldsymbol{r}(\boldsymbol{t}) = \frac{3}{4}\left(1 - \frac{1}{1+e^{\frac{(0.368\,L)-t}{(0.074\,L)}}}\right) + \frac{1}{2} \tag{A1}$$

Where ***L*** is the total length of the bat and ***t*** is the displacement along the length of the bat. Figure A1 below is a plot of this function for a 34-inch bat.

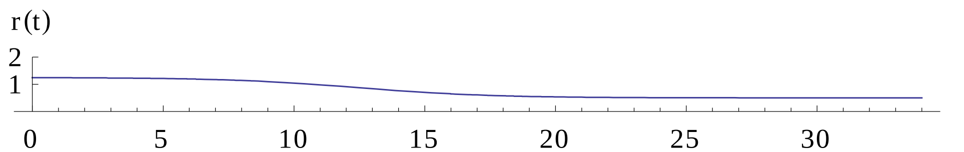

Figure A1. Plot of ***r(t)*** for 34-inch Bat

The next step was to use this function in the relationship between ***k*** and ***r***, namely:

$$\boldsymbol{k} \propto \boldsymbol{r}^4 \tag{A2}$$

Using this relationship, we created a plot that demonstrates the drastic change in the stiffness of the bat as you change the point along the length. This plot (for the same 34-inch example) is given in Figure A2.

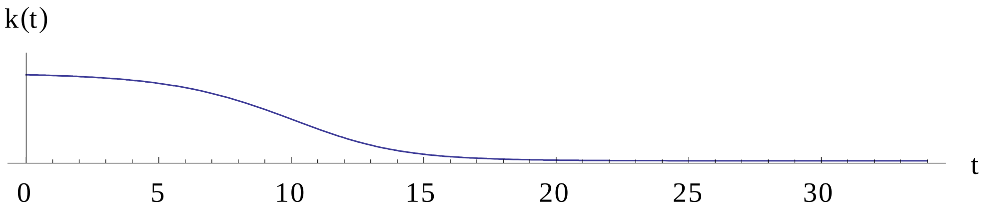

Figure A2. Relative Stiffness for 34-inch Bat

The fourth power function in the relationship between ***k*** and ***r*** is very visible in the rapid drop in the relative ***k*** value as one moves toward the handle of the bat. This explains the fact that the vibration that occurs there has the greatest amplitude, since a lower ***k*** corresponds to a more flexible state, allowing for easier vibration, or less damping.

While this creates a more accurate model for the bending mode vibrations, it does significantly change our predicted sweet spot location.

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